Math 10A
Worksheet, Discussion #3; Wednesday, 6/20/2018
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1 Limits

1.1 Example

1. Compute $\lim_{x\to-\infty} \frac{x+3}{\sqrt{9x^2-5x}}$.

Solution: The largest exponent of x in the denominator is $\sqrt{x^2}$ or $|x|$. Dividing by this value gives us

$$\lim_{x\to-\infty} \frac{x+3}{\sqrt{9x^2-5x}} = \lim_{x\to-\infty} \frac{x/|x|+3/|x|}{\sqrt{9-5/x}} = \frac{-1+0}{\sqrt{9-0}} = \frac{-1}{3}.$$

1.2 Problems

2. Compute $\lim_{x\to\infty} \frac{(2x+5)(x-2)}{(7x-2)(3x+1)}$.

Solution: The largest exponent in the denominator is x^2 and dividing by that gives

$$\lim_{x\to\infty} \frac{(2x+5)(x-2)}{(7x-2)(3x+1)} = \lim_{x\to\infty} \frac{(2+5/x)(1-2/x)}{(7-2/x)(3+1/x)} = \frac{2}{21}.$$

3. None of the following functions are defined at $x = 3$. Is there a way to define each function at $x = 3$ so that it's still continuous?

   (a) $f(x) = \frac{x^2-9}{x-3}$

   (b) $g(x) = \frac{x^2+3x-9}{x-3}$

   (c) $h(x) = \frac{x^2-7x+6}{x-3}$

Solution:

(a) Yes because if $x \neq 3$, then $f(x) = \frac{(x+3)(x-3)}{x-3} = x+3$ so $\lim_{x\to 3} f(x) = 6$ is defined.

(b) No because the limit $\lim_{x\to 3} g(x) = DNE$ since on the left it goes to $-\infty$ and on the right ∞.

(c) No because the limit $\lim_{x\to 3} h(x) = DNE$ since on the left it goes to ∞ and on the right $-\infty$.

4. Compute $\lim_{x\to\infty} \frac{2-\cos(x)}{3-2x}$.

Solution: We use the squeeze theorem which says $-1 \leq \cos(x) \leq 1$ so $1 \leq 2 - \cos(x) \leq 3$ and

$$\lim_{x\to\infty} \frac{1}{3-2x} \leq \lim_{x\to\infty} \frac{2-\cos(x)}{3-2x} \leq \lim_{x\to\infty} \frac{3}{3-2x}.$$

But the first and last limit are by 0 so the middle limit must be 0 too.

5. Find $\lim\limits_{x\to 2} \dfrac{x^2-4}{\sqrt{x}-\sqrt{4-x}}$.

Solution: We multiply the top and bottom by the conjugate to get that

$$\lim_{x\to 2} \frac{x^2-4}{\sqrt{x}-\sqrt{4-x}} = \lim_{x\to 2} \frac{(x^2-4)(\sqrt{x}+\sqrt{4-x})}{x-(4-x)} = \lim_{x\to 2} \frac{(x-2)(x+2)(\sqrt{x}+\sqrt{4-x})}{2(x-2)}$$

$$= \lim_{x\to 2} \frac{(x+2)(\sqrt{x}+\sqrt{4-x})}{2} = \frac{4(\sqrt{2}+\sqrt{2})}{2} = 4\sqrt{2}.$$

2 Vertical Asymptotes

2.1 Example

6. Find $\lim\limits_{x\to 0^-} x^{-1}e^{x^{-2}}$.

Solution: As $x \to 0^+$, we know that $x^{-1} \to -\infty$ and $x^{-2} \to \infty$ so $x^{-1}e^{x^{-2}} \to -\infty \cdot e^{\infty} = -\infty$.

2.2 Problems

7. Find $\lim\limits_{x\to 2\pi^-} x\csc(x)$.

> **Solution:** As $x \to 2\pi^-$, we know that $\sin(x) \to 0^-$ and hence $x\csc(x) = \frac{x}{\sin(x)} \to -\infty$.

8. Find $\lim\limits_{x\to 2^-} \dfrac{x^2-2x-8}{-(x^2-5x+6)}$.

> **Solution:** Factoring, we have that $\dfrac{x^2-2x-8}{-(x^2-5x+6)} = \dfrac{(x-4)(x+2)}{-(x-2)(x-3)}$. As $x \to 2^-$, we get $\frac{-8}{0}$ and so we need to care about the sign. Since $x < 2$, the signs are $\frac{-\cdot+}{-\cdot-\cdot-} = +$ so the limit is $+\infty$.

9. Find $\lim\limits_{x\to 3^+} \dfrac{\sqrt{x}}{(x-3)^4}$.

> **Solution:** As $x \to 3^+$, we have that $(x-3)^4 \to 0^+$ and so the limit is ∞.

2.3 Extra Problems

10. Find $\lim\limits_{x\to 2\pi^+} x\csc(x)$.

> **Solution:** As $x \to 2\pi^+$, we know that $\sin(x) \to 0^+$ and hence $x\csc(x) = \frac{x}{\sin(x)} \to \infty$.

11. Find $\lim\limits_{x\to 2^+} \dfrac{x^2-2x-8}{-(x^2-5x+6)}$.

> **Solution:** Factoring, we have that $\dfrac{x^2-2x-8}{-(x^2-5x+6)} = \dfrac{(x-4)(x+2)}{-(x-2)(x-3)}$. As $x \to 2^+$, we get $\frac{-8}{0}$ and so we need to care about the sign. Since $x > 2$, the signs are $\frac{-\cdot+}{-\cdot+\cdot-} = -$ so the limit is $-\infty$.

12. Find $\lim\limits_{x\to 3^-} \dfrac{\sqrt{x}}{(x-3)^4}$.

Solution: As $x \to 3^-$, we have that $(x-3)^4 \to 0^+$ and so the limit is ∞.